SINGULARITY-FREE COSMOLOGICAL SOLUTIONS IN STRING THEORIES

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Abstract

Singularity-free cosmological solutions may be obtained from the string action at tree level if the dimension of the space-time is greater than 10 and if brane configurations are taken into account. The behaviour of the dilaton field in this case is also regular. Asymptotically a radiative phase is attained indicating a smooth transition to the standard cosmological model.

PACS number(s): 04.20.Cv., 04.20.Me

At tree level, the string action can be reduced to the gravitational action coupled non-minimally to the dilaton field and to gauge fields. These gauge fields may couple non-minimally or minimally to the dilaton field depending if they come from the Neveu-Schwartz or Ramond-Ramond sector of the string action [1]. In general, the dimension of the space-time in string theories is equal to 10 if supersymmetry is considered. Bosonic string may leave in a 26 dimensional space-time. But, M-theory requires a 11-dimensional space-time while the F-theory is generally formulated in 12 dimensions [2, 3, 4].

Cosmology is an arena where the consequences of string theories can be explored, since the typical string effects must manifestate themselves at very large energy levels, which can be attained only in the very early universe. One of the expectation concerning string cosmology relies on the possibility to obtain singularity-free primordial cosmological models. However, this expectation has been frustrated until now: even when a non-singular four dimensional space-time is obtained, divergences in the dilaton field appear and employement of an effective action at tree level becomes doubtfull [5, 6]. In some cases, as the singularity is approached non-linear geometrical terms may be taken into account, avoiding the appearence of divergences in the curvature invariants. But, the entire scenario is composed of many branchs rendering all the model quite artificial [7].

In this work we consider the string effective action in four dimension obtained from the original D-dimensional theory by dimensional reduction and truncation. This effective action will be coupled to ordinary radiative matter, which may be considered as a manifestation of the electromagnetic term existing in the Neveu-Schwartz and/or Ramond-Ramond sector. Singularity-free solutions are found. But, in order to obtain a complete regular model, in the geometric and dilaton fields, the dimension of the space-time must

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be greater than 10. The recent activity with the so-called M-theory and F-theory, as well as with another string-type models in dimensions greater than 10, may render the model developped here attractive. One important ingredient to obtain the regular models found here is to consider p-branes configuration. In particular, eleven and twelve dimensional space-times with a 4-brane are quite interesting models, representing some of the cases where a complete regular scenario is obtained.

Let us consider the string effective action at tree level:

$$L = \sqrt{-\tilde{g}}e^{-\tilde{\sigma}}\biggl\{\tilde{R} - \tilde{\omega}\tilde{\sigma}_{;A}\tilde{\sigma}^{;A} - \frac{1}{12}H_{ABC}H^{ABC}\biggr\} \tag{1}$$

where $\tilde{\sigma}$ is the dilatonic field, H_{ABC} is the axionic field and $\tilde{\omega}$ is the dilatonic coupling constant. The tildes indicate that all quantities are considered in a D-dimensional space-time. In particular, the dilatonic coupling constant, when a p-brane configuration is taken into account is given by [8]

$$\tilde{\omega} = -\biggl\{\frac{(D-1)(p-1)-(p+1)^2}{(D-2)(p-1)-(p+1)^2}\biggr\} \tag{2}$$

where p denotes the order of the brane: $p = 0$ indicates a pointlike configuration, $p = 1$ a stringlike configuration, $p = 2$ a membrane, and so on. When $p = 1$, $\tilde{\omega} = -1$ for any dimension of the space-time.

The D-dimensional metric is written as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu - e^{2\beta}\delta_{ij}dx^i dx^j \quad , \tag{3}$$

where $g_{\mu\nu}$ is the four dimensional metric, e^β is the scale factor of the d-dimensional internal space which we suppose to be homogenous and flat. Hence we obtain the following effective action in four dimension

$$L = \sqrt{-g}\phi\biggl[R - \gamma_{;\rho}\gamma^{;\rho} - \omega\frac{\phi_{;\rho}\phi^{;\rho}}{\phi^2} - \phi^{n-1}\Psi_{;\rho}\Psi^{;\rho}\biggr] + L_m. \tag{4}$$

In this action,

$$\begin{aligned} \phi = e^{d\beta-\tilde{\sigma}} \quad &, \quad \gamma = a\beta + b\ln\phi \quad , \\ a = \{d(d+1)+\tilde{\omega}d^2\}^{1/2} \quad &, \quad b = -d(1+\tilde{\omega})\{d(d+1)+\tilde{\omega}d^2\}^{-1/2} \quad . \end{aligned} \tag{5}$$

The field Ψ comes from the axionic term. The parameter n was included in order to consider other effective actions, for example, those coming from supergravities or pure multidimensional space-time [5]. The string case with an axionic field corresponds to $n = -1$. The term L_m represents the ordinary matter. Here, we will be interested in the radiative fluid only, since it seems more realistic when we have in mind the primordial Universe. Moreover, a radiative fluid may be obtained through the reduction of string action to four dimensions. For example, the 5-form existing in the Ramond-Ramond sector of the superstring type IIB leads in four dimension to, besides other scalar fields, an eletromagnetic field without coupling with the dilaton or moduli fields. This is a general

feature of $D/2$-forms in a D-dimensional space-time [9]. The new coupling constant is given by

$$\omega = -\left\{\frac{(d-1)\tilde{\omega}+d}{(d+1)\tilde{\omega}+1}\right\} \quad . \tag{6}$$

Notice that, when $\tilde{\omega} = -1$, $\omega = -1$. Hence, the pure string case is a fixed point.

From (4) we obtain the field equations:

$$\begin{aligned}
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= 8\pi\frac{T}{\phi} + \frac{\omega}{\phi^2}\Big(\phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\rho}\phi^{\rho}\Big) + \frac{1}{\phi}\Big(\phi_{\mu\nu} - g_{\mu\nu}\Box\phi\Big) + \\
&+ \Big(\gamma_{;\mu}\gamma_{;\nu} - \frac{1}{2}g_{\mu\nu}\gamma_{;\rho}\gamma^{;\rho}\Big) + \phi^{n-1}\Big(\Psi_{;\mu}\Psi_{;\nu} - \frac{1}{2}g_{\mu\nu}\Psi_{;\rho}\Psi^{;\rho}\Big) \quad ; &(7)\\
\Box\phi + \frac{1-n}{3+2\omega}\phi^n\Psi_{;\rho}\Psi^{;\rho} &= \frac{8\pi}{3+2\omega}T \quad ; &(8)\\
\Box\Psi + n\Psi_{\rho}\frac{\phi^{;\rho}}{\phi} &= 0 \quad ; &(9)\\
\Box\gamma + \gamma_{;\rho}\frac{\phi^{;\rho}}{\phi} &= 0 \quad ; &(10)\\
T^{\mu\nu}{}_{;\mu} &= 0 \quad . &(11)
\end{aligned}$$

Considering the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a(t)^2\Big(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big) \quad , \tag{12}$$

k being the curvature of the spatial section ($k = 0, 1, -1$ for a flat, close and opened model, respectively), the field equations reduce to the following equations of motion:

$$\begin{aligned}
3\Big(\frac{\dot{a}}{a}\Big)^2 + 3\frac{k}{a^2} &= 8\pi\frac{\rho}{\phi} + \frac{\omega}{2}\Big(\frac{\dot{\phi}}{\phi}\Big)^2 - 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{\dot{\gamma}^2}{2} + \phi^{n-1}\frac{\dot{\Psi}^2}{2} \quad ; &(13)\\
\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{1-n}{3+2\omega}\phi^n\dot{\Psi}^2 &= \frac{8\pi}{3+2\omega}(\rho - 3p) \quad ; &(14)\\
\ddot{\Psi} + 3\frac{\dot{a}}{a}\dot{\Psi} + n\dot{\Psi}\frac{\dot{\phi}}{\phi} &= 0 \quad ; &(15)\\
\ddot{\gamma} + 3\frac{\dot{a}}{a}\dot{\gamma} + \dot{\gamma}\frac{\dot{\phi}}{\phi} &= 0 \quad ; &(16)\\
\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) &= 0 \quad . &(17)
\end{aligned}$$

In these expression, ρ is the density of ordinary matter and p is the pressure which obeys a barotropic equation of state, $p = \alpha\rho$.

The equations (15,16,17) admit the first integrals:

$$\dot{\Psi} = \frac{A}{a^3\phi^n} \quad , \quad \dot{\gamma} = \frac{B}{a^3\phi} \quad , \quad \rho = \rho_0 a^{3(1+\alpha)} \quad , \tag{18}$$

where A, B and ρ_0 are integration constants. Let us now specialize the equations for the radiative fluid case ($\alpha = 1/3$). Equation (14) becomes

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{1-n}{3+2\omega}\frac{A^2}{a^6\phi^n} = 0 \tag{19}$$

which can be reduced to the integral

$$\int \frac{d\phi}{\sqrt{1 - \frac{2A^2}{(3+2\omega)C}\phi^{1-n}}} = \sqrt{C}\theta \tag{20}$$

where C is another integration constant, and θ is a new time parameter such that $dt = a^3 d\theta$.

This integral can be explicitly solved. The case $\gamma =$ constant and $\omega > -3/2$ has been studied in [5]. Bouncing solutions, with no curvature singularity, were obtained when $-3/2 < \omega < -4/3$. In one asymptotic the dilaton field takes a constant value; but in the other asymptotic it diverges, leading to a divergence in the string coupling parameter, $g_s = \phi^{-1}$. This fact may render the effective action senseless. Perhaps this problem may be coped with through the duality properties of string theory. But, it is not sure that such duality properties can be applied to the background defined by the solution found in [5]. Here we exploit the possibility that $\omega < -3/2$. In this case the integral (20) can be solved through the redefinition of time parameter. The solution for ϕ takes the form,

$$\phi = \phi_0(\sinh\xi)^{2/(1-n)} \quad , \quad \phi_0 = \left\{\frac{(3+2\omega)C}{2A^2}\right\}^{1/(1-n)} \quad . \tag{21}$$

In this expression, ξ is the time parameter connected with the cosmic time by

$$dt = \frac{2}{(1-n)\sqrt{C}}\phi_0 a^3(\sinh\xi)^{(1+n)/(1-n)}d\xi \quad . \tag{22}$$

In order to solve (13) and determine the behaviour of the scale factor, we write $a = \phi^{-1/2}b$, rewrite the resulting expression in terms of the time parameter ξ, obtaining the relation

$$\frac{b'}{b} = \pm\sqrt{Mb^2 - kb^4 + B^2 - 1}\frac{\sqrt{|3+2\omega|}}{1-n}\frac{1}{\sinh\xi} \tag{23}$$

where the prime means derivative with respect to ξ. Let us consider the case where $k = 0$ and $B = 0$. This implies a flat universe where, in principle, the internal scale factor is constant. However, notice the it can correspond to a time-dependent internal scale factor which bears a specific relation with the field ϕ. In any case, the scale factor of the external space takes the form

$$a = -a_0[(\sinh\xi)^{-1/(1-n)}] \times \frac{1}{\cos[\ln(\tanh^r\xi/2)]} \quad , \quad r = \pm\frac{\sqrt{|3+2\omega|}}{1-n} \quad . \tag{24}$$

This solution represents a bouncing universe in the interval

$$\xi_i < \xi < x_f \quad , \quad \xi_{i,f} = \ln\biggl[\frac{1+s_{i,f}}{1-s_{i,f}}\biggr] \quad ,$$
$$s_{i,f} = e^{(n_{i,f}+1/2)\frac{\pi}{r}} \quad , \quad n_i = n_f - 1 \quad , \quad n_f < 0 \quad . \tag{25}$$

The cosmic time, in this interval, is such that $-\infty < t < \infty$. This assures the absence of any curvature singularity. Moreover, the field ϕ takes finite non-zero values during all the evolution of the universe. Hence, there is no divergence of the string coupling parameter, even in the extreme of the interval. There is also no divergence in the scale factor (what is trivial if we consider the constant value, of course). So, the solution described by (24) is a complete regular solution. In both asymptotic the scale factor behaves as $a \propto t^{1/2}$ what allows for a smooth transition to a radiative phase and consequently to the standard cosmological model.

The most important difference beteewn the results found here and those presented in [6] is the absence of a divergence in the string perturbative coupling parameter in the initial asymptotic. This is due to the presence of a radiative fluid, a case not considered in that work. It must also be stressed that in [6] the dynamics of the internal space was considered in its full complexity, while here just some particular cases were exploited. But, as it has been already remarked in [6], the possibility to obtain singularity-free solutions arises only if the dimension of the space-time is greater than 10, since anomolous theories are obtained in the Einstein-frame formulation of the effective action.

In fact, the question which arises now concerns if this singularity-free solution can be implemented in the context of string theories. One of the essential features of the model exposed above is that $\omega < -3/2$. This can be only achieved if the dimension of the space-time is such that $D > 10$ for some brane configurations. It excludes the traditional superstring case. However, one example where it can be obtained is in the realm of the so-called M-theory and F-theory, which are formulated in $D = 11$ and $D = 12$ respectively. Both the M-theory and F-theory are connected with string theories, since the superstring models may be viewed as specific background configurations of those higher dimensional theories [2, 10]. For example, for the case $B = 0$, with $D = 11$, $D = 12$ with $p = 4$ we obtain $\omega = \tilde{\omega} = -5/2$ and $-8/5$, respectively. These are interesting situations because one can relate them with the brane world program [11] by which we live in a four-dimensional brane. It must also be stressed that the regular solutions displayed here are valid for $n = -1$, which is the typical value of this parameter for string theories. Notice, however, that in the present letter we have exploited only one of the simplest case. But, our goal here is to show that a complete singularity-free solution, even in the dilatonic field, may be obtained in the context of the string action if we allow the dimension of the space-time be greater than 10. A more complete analysis may reveal other possible situations. Finally, it must be remarked that the gravitational coupling is connected with the inverse of the field ϕ, which takes constant values in both asymptotics in such a way that the value of the gravitational coupling in the first asymptotic is greater than its value in the second asymptotic, what opens the possibility to solve the hierarchial problem of the cosmological constant in a way similar to the the brane cosmology program. In the case treated here, the gravitational coupling dicreases its value by a factor of 10^6 between the initial and

final asymptotics when $\omega = -8/6$ ($D = 12$, $p = 4$).
Acknowledgement: We thank CNPq (Brazil) for partial financial support.

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